

Problem Set 5: Hypothesis Testing

1. For a sample of data Y_1, Y_2, \dots, Y_n on a random variable $Y \sim IN(\mu, \sigma^2)$.
 - (a) You want to test the hypothesis that μ equals a specified value μ_o . How would you test this hypothesis?
 - (b) Explain Type I and Type II errors. How did you deal with these two types of error in your answer to (a)?
 - (c) Suppose that you wanted to calculate the probability of observing a value of Y greater than a specified value Y_0 . How would you do this?
 - (d) Assume that $\hat{\mu}$, the sample mean, is an estimator for μ . How would you calculate the 95% confidence interval?
2. Suppose that you have a machine for filling 1 kilogram bags of sugar. The machine is set so that the weight of a bag should be a normally distributed random variable with an expected value of 1005g and a standard deviation of 2g. For a random variable Z with a standard normal, the cumulative probabilities are:

z	0	0.33	0.5	1	1.5	2	2.5
$P(Z < z)$	0.5	0.6293	0.6915	0.8413	0.9332	0.9772	0.9938

 - (a) What is the probability that a bag of sugar will weight (i) less than 1000g (ii) between 1004g and 1006g?
 - (b) You take a sample of 16 bags of sugar, which have a mean of 1004g. What is (i) the standard error of the mean and (ii) the probability of getting a sample mean of 1004g or less?
 - (c) On the basis of this estimate of the mean would you conclude that the machine was still working correctly?
3. A random sample of 100 post office saving accounts showed the mean value to be 57 with a sample standard deviation of 2. Test the one-sided and two-sided hypotheses that the mean size of post office accounts is 60 at the levels of significance of 5% and 1%.