

**Problem Set 4: Estimation and Confidence Intervals**

1. Suppose you have a sample of data,  $Y_i, i = 1, 2, \dots, n$ , where

$$Y \sim IN(\mu, \sigma^2)$$

- (a) Explain what  $Y \sim IN(\mu, \sigma^2)$  means.
  - (b) How would you obtain unbiased estimates of  $\mu$  and  $\sigma^2$ ? Explain what unbiased means.
  - (c) How would you estimate the standard error of your estimate of  $\mu$ ?
  - (d) Suppose that the distribution of your sample was not normal but highly skewed. Explain what this means and discuss what other measures of central tendency that you might use.
2. Let  $X_1, X_2, \dots, X_n$  be a random sample from a population with known variance  $\sigma^2$ . The following are three possible estimators for the population mean,  $\mu$ .

$$\begin{aligned}\hat{\mu} &= \frac{\sum X_i}{n} \\ \tilde{\mu} &= \frac{\sum X_i}{n+1} \\ \check{\mu} &= X_1\end{aligned}$$

- (a) Calculate the expected value and variance of each estimator.
  - (b) Investigate the properties of the estimators with regards to unbiasedness and minimum variance.
3. A random sample of 100 record shops found that the average weekly sale of a particular record was 260 copies, with a standard deviation of 96. Find the point and the 95% interval estimates for the true population mean. Explain why an interval estimate is better than a point estimate. What factors determine the width of a confidence interval?