

Problem Set 3: Probability Models and Distributions

1. If X and Y are random variables with $E(X) = \mu_X$, $Var(X) = \sigma_X$ and $E(Y) = \mu_Y$, $Var(Y) = \sigma_Y$, and a, b are constants show that:

(a) $Var(aX - bY) = a^2Var(X) + b^2Var(Y) - 2abCov(X, Y)$

(b) if X, Y are independent, $Cov(X, Y) = 0$

2. Marks on an exam are normally distributed with expected value 50 and standard deviation 10.

- (a) What proportion of the students get

(i) < 30 ;

(ii) between 30 and 50;

(iii) over 50.

- (b) What mark does a student need to get into the top 15%.

3. Calculate the following:

- (a) If X has a normal distribution with expected value 1 and standard deviation 2, find the following probabilities:

i) $P(|X| > 2)$ ii) $P(X < 3 | X \geq 0)$

- (b) Let Y have a t distribution with 20 degrees of freedom: find two numbers a and b such that

$$P(a < Y < b) = 0.80$$

- (c) Let W have an F distribution with 3 and 10 degrees of freedom: find a number b such that

$$P(W > b) = 0.05$$