School of Oriental and African Studies **University of London**

Department of Economics

Preliminary Statistics **September 2013**

Problem Set 2: Probability Theory

1. Define $P(A)$ as the probability of event A happening; $P(B)$ the probability of event B happening; $P(A \cap B)$ the probability of both A and B happening; $P(A \cup B)$ the probability of either A or B happening; and $P(A \mid B)$ the probability of A happening conditional on B already having happened.

   (a) What is $P(A \cap B)$ if A and B are mutually exclusive.

   (b) What is $P(A \cap B)$ if A and B are independent?

   (c) What is $P(A \cup B)$?

   (d) Show that
   $$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

2. Consider the following game. A fair coin is tossed until it comes up heads and you get paid £(2^n) if it comes up heads on the n-th throw. If it comes up heads the first time you get £2 and the game stops. If it comes up heads, for the first time on the second throw you get £4=$(2)^2$ and the game stops; and so on. What is the expected value of this game? How much would you personally pay to play it?

3. From a large survey of customers using a chain of coffee shops, 60% of the customers are male, 50% purchase food, 15% are both Male and purchase food. What are the probabilities that:

   (a) A customer is both female and purchases food?

   (b) A female customer purchases food?

   (c) A customer purchasing food is female?

4. Students take two exams A and B. 60% pass A, 80% pass B, 50% pass both.

   (a) Fill in the remaining five elements of the joint and marginal distributions below, where PA indicates pass A, FB fail B, etc.

   (b) What is the probability of a student passing exam B given that he/she passed exam A?

   (c) Are the two events passing A and passing B

i. mutually exclusive

ii. independent?

	PA	FA	B
PB	50		80
FB			
A	60		100

5. **(Optional)** You are in a US quiz show. The host shows you three closed boxes in one of which there is a prize. The host knows which box the prize is in, you do not. You choose a box. The host then opens another box, not the one you chose, and shows that it is empty. He can always do this. You can either stick with the box you originally chose or change to the other unopened box. What should you do: stick or change? What is the probability that the prize is in the other unopened box?