Preliminary Statistics September 2012

Example Probability Theory

Question

Let A be the event that a student is enrolled in an Accounting course, and let S be the event that a student is enrolled in a Statistics course. It is known that 30% of all students are enrolled in an Accounting course. 40% of all students are enrolled in Statistics, and 15% are enrolled in both Accounting and Statistics.

(i) What is the probability that a student is not enrolled in any of the two courses; neither in Accounting, nor in Statistics?
(ii) What is the probability that a student enrolled in Accounting will also be enrolled in Statistics?
(iii) Are the two events: "enrolled in Accounting" and "enrolled in Statistics"
  a. Mutually exclusive?
  b. Independent?
(iv) Can two events be both mutually exclusive and independent?

Answer

What information is given in the text?

1. $P(A) = 0.3$ i.e. the marginal probability of being enrolled in Accounting is 0.3.
2. $P(S) = 0.4$ i.e. the marginal probability of being enrolled in Statistics is 0.4.
3. $P(A \cap S) = 0.15$ i.e. the joint probability of being (simultaneously) enrolled in Accounting and Statistics is 0.15.

(i) The first questions asks for the joint probability of not A and not S, i.e. $P(\text{not } A \cap \text{not } S)$?
  a. This is basically all students other than those in $P(A \cap S)$, $P(A)$, and $P(S)$. This means $P(\text{not } A \cap \text{not } S) = 1 – [P(A) + P(S) – P(A \cap S)] = 0.4$.
(ii) The second question asks for the conditional probability of a student being enrolled in Statistics given that she is already enrolled in accounting, i.e. $P(S|A)$?
  a. Here you can use the formula: $P(S|A) = P(A \cap S)/P(A) = 0.15/0.3 = 0.5$.
(iii) You would need to know $P(A \cap S)$ and $P(S|A)$.
  a. If the two event were mutually exclusive, then $P(A \cap S) = 0$ per definition. However, we know $P(A \cap S) = 0.15$, so the two events are not mutually exclusive.
  b. If the two events were independent, than the conditional probability of A given S would have to equal the marginal probability of A. However, $P(S|A) = 0.5$ and $P(A) = 0.3$ so that $P(S|A) \neq P(A)$ and the two events are not independent.
(iv) Independence requires that the occurrence of one event does not change the probability of the other event occurring. However, per definition, if two events are mutually exclusive event A happening leads to the probability of event B happening to be zero regardless of the marginal probability of event B.