Preliminary Stats September 2012

Excurse: Marginal, Joint, and Conditional Probabilities

The concept of marginal, joint, and conditional probability density functions can be understood in terms of general probability theory.

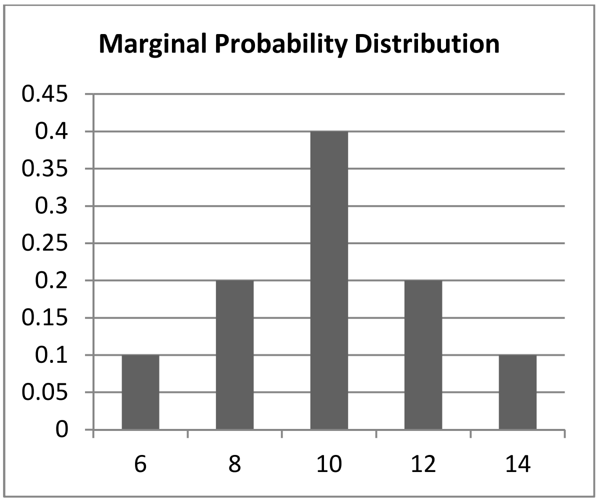
The marginal probability density function is a functional form which ascribes a probability to each possible realisation of a random variable [X].

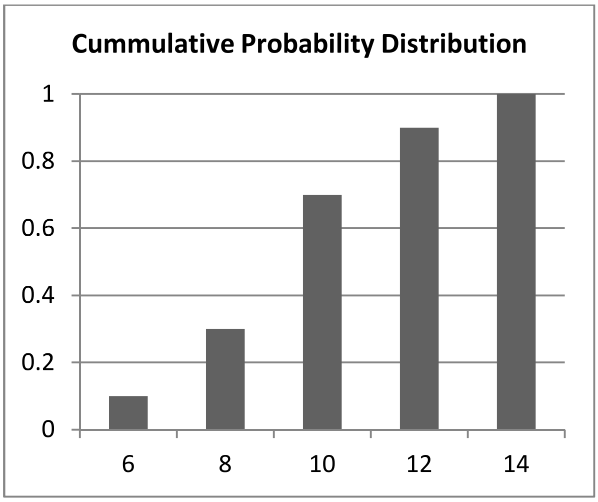
$$P(X = x_i) = f(x_i)$$

Hence for every realisation x_i of X there is a probability $f(x_i)$.

Assuming that X can take on the below values with the ascribed probabilities the marginal probability function would provide the probability for each of the events happening (discrete case).

x_i	6	8	10	12	14
$f(x_i)$.	0.1	0.2	0.4	0.2	0.1





The joint probability density function is the functional form which ascribes a probability to each event where two particular realisations of the two random variables X and Y happening simultaneously.

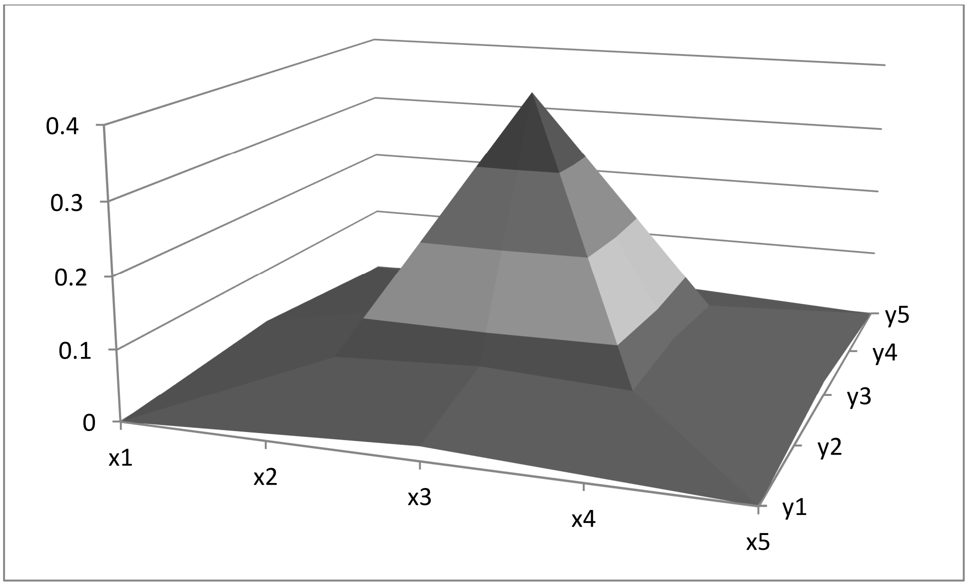
$$P(X = x_i \cap Y = y_i) = f(x_i, y_i)$$

Hence for every (joint) pair of simultaneous realisations of X and Y possible there is a probability ascribed to. Assuming that the joint probabilities of each of the 25 pairs of the five possible realisations of X and Y respectively would be as following:

	x_1	x_2	x_3	x_4	x_5
y_1	0.00	0.01	0.02	0.01	0.00
y_2	0.01	0.05	0.06	0.05	0.01
y_3	0.02	0.06	0.40	0.06	0.02
y_4	0.01	0.05	0.06	0.05	0.01

y_5	0.00	0.01	0.02	0.01	0.00

Then the joint probability distribution would look like:



The conditional probability density function is the functional form which ascribes a probability of every realisation of a random variable X given a certain realisation of Y.

$$P(X = x_i | Y = y_i) = f(x_i | y_i)$$

Given the definition of conditional probabilities we know:

$$P(X = x_i | Y = y_i) = \frac{P(X = x_i \cap Y = y_i)}{P(Y = y_i)} = \frac{f(x_i, y_i)}{f(y_i)}$$

Hence, based on the joint probability function and marginal probability function we can calculate the conditional probability function. Again if X and Y are two independent random variables the conditional probability function of X given Y would equal the marginal probability function of X.

NOTE 1: In the probability theory lecture we were working with exactly the same principles, however, analysing only one realisation or in the joint and conditional case two realisations of a random variable.

NOTE 2: The same can be done with continuous random variables. However, numerical examples are difficult as by definition we cannot take all possible realisations into considerations as there is an infinite amount of them.