Preliminary Statistics September 2012

Constants, Random Variables, and Expected Values

Assume some advertisement campaign for Tesco grocery. As a customer you can participate by buying a voucher for £10. There are two different vouchers between which you can choose: one gives you an additional 20% to the value of the voucher (so you get a voucher over £12 by paying £10) and the other gives one of the following: -20%, -40%,100%,+20%, or +40% . The vouchers for the second option are sealed and you do not know which value you get when choosing one. However, you know that there are 1,000 vouchers of each option soled in each store, with the following composition:

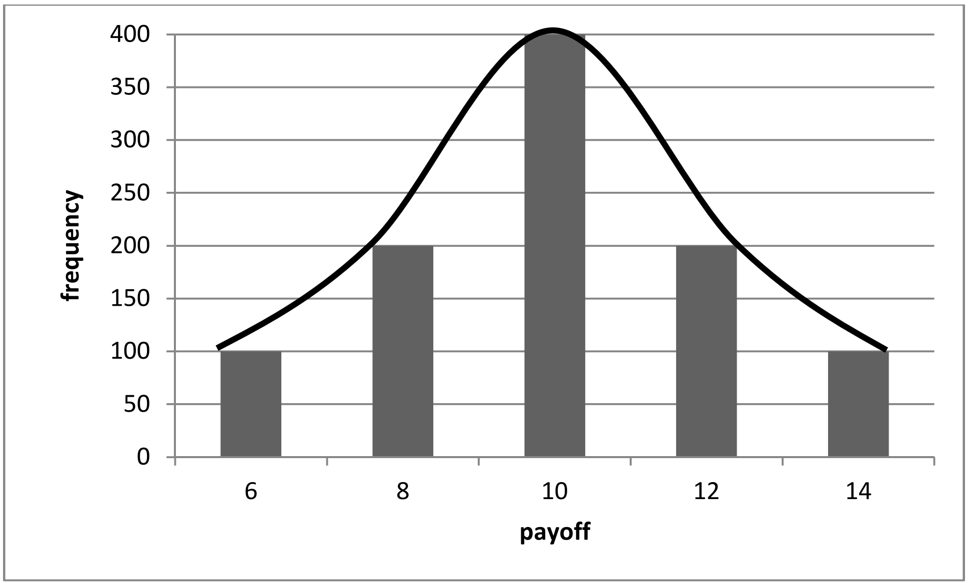
Option 1	Quantity	Prob.	Option 2	Quantity	Prob.
12	1,000	1	6	100	0.1
			8	200	0.2
			10	400	0.4
			12	200	0.2
			14	100	0.1
$E(O_1) = 12 * P(12) = 12$			$E(O_2) = \sum pay_i * P(pay_i) = (0.1 * 6 + 0.2 * 8 + 0.4 * 10 + 0.2 * 12 + 0.1 * 14) = 10$		

The first option is certain hence the event of taking the first option does not have a random outcome but a fixed one. The expected value is always equal the certain outcome. In this sense it is a constant and not a random variable.

The first option provides an uncertain outcome. However, the expected payoff from choosing this option would be £10, which is the arithmetic mean (you can estimate this by either $E(O_2) = \frac{\sum pay_i * quantity_i}{N}$ or by $E(O_2) = \sum pay_i * P(pay_i)$. These two are exactly the same.

Hence, the expected value of a random variable is the sum of its possible outcomes weighted by their respective probabilities (or the sum of the possible outcomes multiplied by the frequency with which they are occurring divided by all outcomes).

Graphically:



The expected value for discrete random variables (as in the example) is given by:

$$E(O_2) = \sum pay_i f(pay_i) = \mu,$$

$$with\ f(pay_i) being\ the\ respective\ probabilities\ for\ each\ pay_i\ (e.g. 0.1\ for\ 6)$$

The expected value for continuous random variables (assuming that the payoffs would be continuous and not discrete) would be:

$$E(O_2) = \int_{-\infty}^{\infty} pay_i f(pay_i) dPay = \mu$$

In a similar manner one can describe the probability of getting a higher or equal pay then the price paid for participating i.e. 10:

Discrete case:

$$P(Pay \geq 10) = \sum_{i=10} f(pay_i) = f(pay_{10}) + f(pay_{12}) + f(pay_{14}) = 0.4 + 0.2 + 0.1 = 0.7$$

Continuous case:

$$P(Pay \geq 10) = \int_{10}^{\infty} f(pay_i) dPay$$